Technical Comments

Comment on "Investigation of the AP Composite Solid-Propellant Deflagration Mechanism by Means of Experimental Analog Techniques"

J. BURGER*
Institut Français du Pétrole, Rueil Malmaison, France

IN the paper of Ref. 1, the authors make some remarks on the work performed at the University of Louvain applying the porous plug burner technique.[2] The authors compare two methods of cementing the AP particles, either by humid wetting or by washing with a solution of 3 g PMM in 100 ml acetone. They conclude first that the grains cemented by the first method expell small particles when submitted to methane throughput rates larger than those corresponding to stoichiometric combustion. Furthermore, the authors point out that the results obtained with grains cemented by the second method are influenced by the presence of PMM binder. The writer agrees with this observation provided a non-negligible amount of PMM has been introduced in the cementing method. However, in our own work, a solution of 4 g PMM in 1000 ml acetone has been used (7.5 times less concentrated); in these conditions we believe that the influence of PMM cement is negligible;[2] still, we did not observe a noticeable expulsion of AP particles.

As to our working hypothesis about the flame stoichiometry, it seems to be also the choice of the authors themselves since they make the same assumption as ours about the AP decomposition, when calculating the equivalence ratio for PS-AP or PMM-AP loose granular mixtures. At atmospheric pressure we have observed the maximum regression rate at an equivalence ratio of one, in quite a lot of systems: CH_4/AP; C_3H_8/AP; neo C_5H_{12}/AP; C_2H_4/AP; H_2/AP; MMM/AP; PMM/AP, etc. From these observations, in the same way as the authors did from their observations on the PMM/AP system, we considered this as a strong argument in favor of the "one stage" flame model. In one single case, namely that of the NH_3/AP system, the maximum regression rate corresponded to 0.4 equivalence ratio and afterburning was observed at the top of the tube.[3]

Further considerations were emphasized[2] about the structure of the one-stage flames; these are to be considered as nearly perfect premixed flames, provided the dead space between the regressing surface and the flame zone remains smaller than the diffusion distance between fuel and oxidizer. Elementary theoretical equations lead to the introduction of a dimensionless number $D_i = a\dot{m}/\rho\mathfrak{D}$ (a = radius of the solid particles; $\dot{m}$ = total mass burning velocity; ρ = density and $\mathfrak{D}$ = interdiffusion coefficient in the gas phase). We consider that the flame is premixed when $D_i \leqslant 8$. In the case of conventional fuels, the following criterion can be used: $a\dot{m} \leqslant 10^{-3}$ g·cm^{-1}s^{-1}. At atmospheric pressure $\dot{m}$ is generally of the order of magnitude of 10^{-1} g·cm^{-1}s^{-1} and the hypothesis of premixed flames would be valid if the particle size (2a) remains lower than 200 μ. Our, and the author's, experiment were performed with AP particle size of 250-500 μ, and the premixed model is a fairly good approximation. A further study on bundles of microjet flames is in progress at the University of Louvain[4] applying the analogical burner technique.[5]

Received October 24, 1968.
* Research Associate.

References

[1] McAlevy, R. F., III et al., "Investigation of the AP Composite-Solid-Propellant Deflagration Mechanism by Means of Experimental Analog Techniques," *AIAA Journal*, Vol. 6, No. 7, July 1968, pp. 1243–1251.

[2] Burger, J., "Contribution à l'Étude de la Déflagration des Propergols Hétérogènes," *Revue de l'Institut Français du Pétrole et Annales des Combustibles Liquides*, Vol. XX-9, 1965, pp. 1–52; also Transl. 1986, Bureau of Naval Weapons.

[3] Burger, J. and Van Tiggelen, A., "Étude de la Combustion de Propergols Hybrides et Composites," *Bulletin de la Société Chimique de France*, 1964, pp. 3122–3130.

[4] Lys, X., "Étude Analogique du Mécanisme de la Déflagration des Propergols Solides," thesis, Louvain, 1968.

[5] Burger, J., Van Tiggelen, A., and Poncelet, J., "Technique du Brûleur Analogique en Vue d'une Application aux Propergols Solides," *Astronautica Acta*, Vol. II, 1965, pp. 57–34.

Reply by Authors to J. Burger

R. F. MCALEVY III* AND R. B. COLE†
Stevens Institute of Technology, Hoboken, N. J.

BURGER'S comments deal with differences between the findings from this laboratory and his concerning 1) the conditions at which our respective test specimens lose structural integrity and expel unburned AP particles, and 2) conclusions regarding the stoichiometry of AP porous-plug burners.

1) Several questions still remain in this area. Our method of cementing was apparently very close to Burger's, yet we have found that plastic concentrations (in the cement) much higher than his are required to insure structural integrity of the cemented porous plugs. Contrarily, while we have had success in using water AP solution-bonded specimens in a limited range of operation (without loss of structural integrity), Burger apparently has observed expulsion of unburned AP particles from such plugs under similar operating conditions (Ref. 1, p. 12). These differences may be due either to differences in detailed procedures for specimen preparation or to differences in experimental resolution of the threshold for losing structural integrity. These differences might be resolved by an exchange of specimens and/or detailed descriptions of preparation techniques or by more extended investigations of influences of such techniques on combustion behavior.

2) We have never chosen to use or present the nominal, over-all reaction stoichiometry hypothesized by Burger as anything more than an arbitrary, but convenient, basis for normalizing values of $\dot{m}_f/\dot{m}_0$. Burger's view apparently has been that this stoichiometry is "justified" by a regression rate maximum near $\phi = 1$ (based on this stoichiometry)

Received December 23, 1968.
* Director, Combustion Laboratory. Associate Fellow AIAA.
† Instructor, Combustion Laboratory. Associate Member AIAA.

(Ref. 1, p. 24). We cannot share this view because of a) the aforementioned uncertainty of deductions based on cemented-burner data, b) our inability to discern an obvious maximum using uncemented burners, and c) the afterburning of fuel gas under conditions that should be lean according to this hypothesized stoichiometry. The fact that Burger failed to observe afterburning with methane (under conditions similar to those for which we observed it) is still, however, troublesome. Current investigations at our laboratory are aimed at resolving such questions of stoichiometry.

References

[1] Burger, J., "Contribution â l'Étude de la Déflagration des Propergols Hétérogènes," *Revue Institut Français Pétrole et Annales des Combustibles Liquides*, Vol. XX-9, 1965, pp. 1–52; also Transl. No. 1986, Bureau of Naval Weapons.

Comment on "Large-Amplitude Transverse Instability in Rocket Motors"

John M. Bonnell*
Pratt & Whitney Aircraft, East Hartford, Conn.

The coupling of longitudinal and transverse modes of high-frequency combustion instability was treated theoretically by Temkin[1,2] in an effort to explain the experimental observations of Crump and Price.[3] The object of this comment is to point out that mode coupling is not unique to solid-propellant rocket combustors, but also has been observed in gas-burning rockets.

Osborn and Bonnell[4,5] varied the cylindrical geometry of premixed gas rocket combustion chambers to determine the effect on high-frequency combustion pressure oscillations. It was found that when the length/diameter ratio was such that the frequencies of the fundamental longitudinal and tangential modes were approximately the same, the amplitudes of oscillation increased significantly. There was an accompanying "shift" of the instability region (on a plane of equivalence ratio vs combustion pressure) to a lower combustion pressure. The changes in the wave shapes of the pressure oscillations observed on an oscilloscope confirmed the belief that an interplay between the two aforementioned modes was occurring. The net effect of mode coupling in a premixed gas rocket was an increase in the severity of combustion instability.

References

[1] Temkin, S., "Large-Amplitude Transverse Instability in Rocket Motors," *AIAA Journal*, Vol. 6, No. 6, June 1968, pp. 1202–1204.

[2] Temkin, S., "Mode Coupling in Solid Propellant Rocket Motors," *AIAA Journal*, Vol. 6, No. 3, March 1968, pp. 560–561.

[3] Crump, J. E. and Price, E. W., " 'Catastrophic' Changes in Burning Rate of Solid Propellants During Combustion Instability," *ARS Journal*, Vol. 30, No. 7, July 1960, pp. 705–707.

[4] Osborn, J. R. and Bonnell, J. M., "Importance of Combustion Chamber Geometry in High Frequency Oscillations in Rocket Motors," *ARS Journal*, Vol. 31, No. 4, April 1961, pp. 482–485.

[4] Osborn, J. R. and Bonnell, J. M., "An Experimental Investigation of Transverse Mode Combustion Oscillations in Premixed Gaseous Bipropellant Rocket Motors," Rept. I-60-1, Jan. 1960, Purdue Jet Propulsion Center, Lafayette, Ind.

Received January 2, 1969.

* Senior Assistant Project Engineer, Combustion Group, Engineering Department. Member AIAA.

Systems of First-Order, Nonlinear Differential Equations Convertible to Classical Forms

B. V. Dasarathy*
Southern Methodist University, Dallas, Texas

In a recent note, Mason[1] presented classes of first-order systems reducible to Bernoulli and Ricatti type of equations. The material in his note is a particular case of the work reported earlier in Ref. 2, a relevant extract of which is being presented here.

A general first-order nonlinear system can be described by the differential equation

$$\dot{x} + g(x,t) = 0 \tag{1}$$

(dot denotes differentiation with respect to t).

The most general transformation law involving both the dependent and independent variables (unlike the transformation considered by Mason[1] involving the dependent variable only) can be written as

$$X = X(x,t) \tag{2}$$

$$T = T(x,t) \tag{3}$$

Differentiation of Eqs. (2) and (3) with respect to t gives

$$X' = (X_x\dot{x} + X_t)/(T_x\dot{x} + T_t) \tag{4}$$

where the subscripts x and t denote the corresponding partial derivatives and prime denotes differentiation with respect to T.

Substituting Eq. (1) in Eq. (4),

$$X' = (X_x g - X_t)/(T_x g - T_t) \tag{5}$$

For Eq. (5), which represents the given system in the transformed plane, to be amenable to existing methods of analysis, it should conform to one of the classical forms, such as

Bernoulli type:

$$X' = C_1(T)X^n - C_2(T)X \quad (n \neq 1) \tag{6a}$$

($n = 0$ represents the linear type)

Ricatti type:

$$X' = C_1(T)X^2 + C_2(T)X + C_3(T) \tag{6b}$$

Separable type:

$$X' = C_1(T)F(X) \tag{6c}$$

Exact equation type:

$$X' = M(X,T)/N(X,T) \tag{6d}$$

$$X' = M(X,T)/N(X,T) \tag{6e}$$

(Here $\partial N/\partial T = \partial M/\partial X$.) Substitution of Eq. (5) into any one of Eqs. (6a–6d) results in a corresponding partial-differential equation. The solutions to such partial-differential equations represent the necessary transformation functions for a given first-order system, Eq. (1), to be reducible to one of the classical forms listed previously.

Although a unique general solution to these partial-differential equations in (X, T) may not be obtainable for any given function $g(x,t)$, it is useful even if a particular solution to any one of these partial-differential equations is obtained.

Received December 16, 1968. The author thanks M. D. Srinath, Associate Professor, Information and Control Sciences Center, Southern Methodist University, for inviting his attention to Mason's work.

* Visiting Assistant Professor, Information and Control Sciences Center, Institute of Technology.